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COOLING OF A HORIZONTAL CYLINDER OF WATER THROUGH ITS MAXIMUM DENSITY POINT AT 4°C

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Abstract—Free convection inside a horizontal cylinder which was cooled at a constant rate in time was studied. Temperature distributions, boundary-layer velocity profiles, and flow patterns were measured and compared to predictions of a quasi-steady boundary-layer model which was also developed. Of particular interest to this study was the behavior of free convection as the cylinder was cooled through the maximum density point of water. The theoretical model predicts the existence of three quasi-steady modes of convection for some temperatures below 4°C. The reversal of the flow pattern observed experimentally involved a transition from one quasi-steady mode to another.

NOMENCLATURE

- a, diffusivity of water, $k/(\rho C)$ [m^2/s];
- $\mathbf{g}$, acceleration due to gravity [m/s^2];
- k, conductivity of water [W/(s°C)];
- p, coordinate tangent to cylinder wall;
- q, coordinate perpendicular to cylinder wall;
- t, time;
- u, velocity in boundary layer;
- v_c, velocity in core;
- x, vertical coordinate in core;
- y, horizontal coordinate in core;
- C, specific heat of water;
- C_T, constant, $\left(\frac{\nu}{a^3 g}\right)^{1/5} DH^{4/5}$ [$(°C)^{4/5}$];
- D, diameter of cylinder [m];
- F, nondimensional velocity;
- G, nondimensional boundary-layer thickness;
- H, rate of cooling of cylinder wall [°C/h];
- J, nondimensional temperature;
- J_{CL}, nondimensional center line temperature;
- J', $= \mathrm{d}J/\mathrm{d}\xi$;
- J'', $= \mathrm{d}^2J/\mathrm{d}\xi^2$;
- Pr, Prandtl number;
- R, radius of cylinder [m];
- Ra, Rayleigh number, $\frac{\mathbf{g}|\beta|}{a\nu} D^3 \Delta T^*$;
- Ra_b, Rayleigh number based on measured temperatures, $\frac{\mathbf{g}|\beta|}{a\nu} D^3(T_b - T_w)$;
- T, temperature [°C];
- T_{CL}, centerline temperature;
- T_w, wall temperature;
- T_{wi}, initial wall temperature;
- ΔT^*, characteristic temperature difference;
- U^*, characteristic velocity [m/s].

Greek symbols

- β, coefficient of thermal expansion [$°C^{-1}$];
- δ, boundary-layer thickness;
- δ^*, characteristic boundary-layer thickness;
- ε, $1/(GRa^{1/4})$;
- η, q/δ;
- θ, angle measured from bottom of cylinder;
- ν, kinematic viscosity [m^2/s];
- ξ, x/D;
- ρ, density of water;
- ρ_c, density of water in core flow;
- ρ_0, density at 0°C;
- ψ, temperature $T - T_w$;
- ψ_c, value of ψ in core flow.

INTRODUCTION

THE COOLING of water in a horizontal cylinder is a problem of some practical interest in the design of utility systems for northern climates. The phenomenon of heat transfer inside water pipes has a number of complicating and interesting aspects. If there is no net flow through the pipe or cylinder the heat transfer between the water and the cylinder wall occurs by the normal free convection processes. However, free convection in this situation must, by the nature of the problem, be a time dependent phenomenon. That is if the conditions around the cylinder wall are uniform, the wall and the water will have the same temperature and no convection will occur in the steady state. If one is interested in cooling the water to the point at which ice formation occurs the water must pass through its maximum density point near 4°C. This produces an additional interesting complication in the behavior of the free convection process.

A number of studies of free convection inside a horizontal cylinder have been made. Ostrach [1] has reviewed the studies in which free convection is produced by nonuniformity of the wall temperature. The results of these studies are not of direct applicability to the present problem; however, they do indicate the applicability of a boundary layer analysis to this type of problem. Studies have also been made in which wall temperature conditions were varied with time. Evans [2] has studied the transient behavior of a fluid in a cylinder when the wall temperature was suddenly changed. He found that for large times the cooling rate expressed as a Nusselt number could be correlated to a Rayleigh number. Maahs [3] has found similar correlations in an experiment that approximates a constant flux boundary condition. Eckert and Deaver [4] have studied the temperature distribution and to some extent the flow field in a horizontal cylinder with a uniform wall temperature which changes at a constant rate in time. In this case, after an initial transient period, a quasi-steady situation developes in which the temperature difference between the fluid and the wall becomes constant. In this quasi-steady regime Nusselt and Rayleigh numbers were defined and the cooling or heating rates correlated in terms of them. Hauf and Grifull [5] have done more extensive work on the initial transient period that results from a change in boundary conditions. In the truly transient period the Fourier number must be introduced as well as Nusselt and Rayleigh numbers in order to correlate the observed behavior.

The effect on free convection of the maximum in the density of water has also been studied by a number of authors. Merk [6] has analyzed the effects of melting and of the maximum density on free convection from a sphere of ice submerged in water. He solved this problem using an integral boundary-layer approach. The results show a minimum in the heat-transfer coefficient for water temperatures near 4°C. These results, which agree with experimental measurements [7], were associated with an inversion of the flow pattern that occurred at these temperatures. More extensive work has been done on this problem by Vanier and Tien [8, 9]. They produced a more precise analysis which predicts the same decrease in heat transfer at temperatures near 4°C. Watson [10] has obtained a similar result from a numerical calculation of the free convective flow between vertical plates in water near 4°C. A number of studies [11, 12] have been made of free convection in a horizontal water layer where the effects of the maximum density have been included. In these studies attempts have been made to define an effective Rayleigh number which will correlate measured heat-transfer data in the region of 4°C. The application of this Rayleigh number to other geometric configurations is not, however, practical.

THEORY

The experimental problem investigated in this paper involves time dependent free convection inside a cylinder with the effects of the maximum density of water included. To obtain a simple theoretical model with which the experimental results can be compared a number of major simplifications are required.

First the difficulties arising from temporal variations are minimized by assuming that a quasi-steady state exists. That is it is assumed that temperatures can be written

$$T = \psi(x, y) + T_w(t).$$

For the wall temperature variation used in the experiments

$$T_w = T_{wi} - Ht,$$

this assumption can only be strictly valid if fluid properties are not temperature dependent. In the region of the temperature inversion this assumption will not be true.

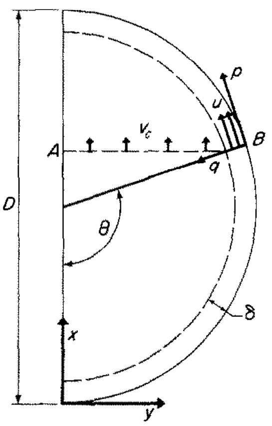


FIG. 1. Coordinates used in quasi-steady model.

The second major assumption made is that the flow can be divided into a core and a boundary layer flow, Fig. 1. A number of approaches may be taken to the calculation of the flow pattern in the cylinder [1, 13, 14]. In this model the simplest possible flow pattern which is qualitatively consistent with experimental observations will be assumed. The velocity in the boundary layer, u, is assumed to be parallel to the local tangent of the wall and confined to a small thickness, δ. Also the velocity in the core, v_c, is assumed to be in the vertical, x, direction and to be independent of the y position. The boundary-layer approximation is valid only for large Rayleigh numbers where $\delta \ll D$. This condition will also be violated for temperatures near the inversion point since β and thus the Rayleigh number becomes small there.

An integral method similar to that used in [6] and [8] for free convection on a vertical surface will be used. The main difference that will exist is that for the case of flow interior to a cylinder the temperature adjacent to the boundary layer, that is in the core, will be a dependent variable which must be determined as a function of position in the cylinder.

For the flow field assumed an integral equation of continuity can be obtained by integrating along the line A–B in Fig. 1. This gives:

$$(R-\delta)\sin\theta v_c + \int_0^{\delta} u\,\mathrm{d}q = 0. \quad (1)$$

The integrated momentum equation in the boundary layer gives:

$$\frac{\partial}{\partial p}\left(\int_0^{\delta} u^2\,\mathrm{d}q\right) = \mathbf{g}\sin\theta \int_0^{\delta} \frac{\rho_c-\rho}{\rho}\,\mathrm{d}q - \nu\left.\frac{\partial u}{\partial q}\right|_{q=0} \quad (2)$$

An energy balance in the core region neglecting conduction terms becomes:

$$v_c \frac{d\psi_c}{dx} = H \quad (3)$$

where ψ_c is the value of ψ in the core flow. It, like v_c, is assumed to be a function only of x. Finally an energy balance in the boundary layer requires that:

$$\frac{\partial}{\partial p}\left(\int_0^{\delta} u\psi \, dq\right) - \psi_c \frac{\partial}{\partial p}\left(\int_0^{\delta} u \, dq\right) - H\delta = -a \frac{\partial \psi}{\partial q}\bigg|_{q=0}. \quad (4)$$

The equations can be normalized using the following characteristic parameters

$$U^* = \frac{a}{D} Ra^{1/2}, \qquad \delta^* = D Ra^{-1/4}$$

and

$$\Delta T^* = \frac{D^2 H}{a} Ra^{-1/4}$$

where the Rayleigh number is defined

$$Ra = \frac{g|\beta|}{a\nu} D^3 \Delta T^* = \left(\frac{g|\beta| H}{a^2 \nu}\right)^{4/5} D^4.$$

The value of β will be defined later. Dimensions are normalized such that

$$\xi = x/D, \qquad \eta = q/\delta,$$

and

$$\delta = \delta^*/G(\xi).$$

Similarity forms are assumed for the velocity and temperature profiles to match the required conditions at $\eta = 0$ and 1. These were

$$u = U^* F(\xi)\eta(1-\eta)^2 + v_c \sin\theta \eta(1+\eta-\eta^2),$$

and

$$\psi = \Delta T^* J(\xi)\eta(2-\eta).$$

The core velocity, v_c, can be eliminated between equations (1) and (3) and the resulting set of equations are:

$$\frac{F}{G}\frac{dJ}{d\xi} = -6\sin\theta(1-\tfrac{5}{6}\varepsilon) \quad (5)$$

$$\frac{1}{Pr} G \frac{\sin\theta}{105} \frac{d}{d\xi}\left(\frac{F^2}{G}\right) = \frac{\sin\theta}{|\beta|\Delta T^*}\int_0^1 \frac{\rho_c - \rho}{\rho} d\eta - G^2 F(1-\tfrac{1}{6}\varepsilon) \quad (6)$$

$$\sin\theta \frac{d}{d\xi}\left[\frac{JF}{G}(1-\tfrac{29}{18}\varepsilon)\right] - \sin\theta J \frac{d}{d\xi}\left[\tfrac{20}{12}\frac{F}{G}(1-\tfrac{7}{6}\varepsilon)\right] + \varepsilon = -40GJ \quad (7)$$

where

$$\varepsilon = 1/(G Ra^{1/4}).$$

Initially the density will be assumed to be a linear function of temperature so that

$$\int_0^1 \frac{\rho_c - \rho}{\rho} d\eta = -\tfrac{1}{3}\beta \Delta T^* J.$$

Then using the assumption of large Prandtl number [6], $Pr \approx 10$ at 4°C, the momentum term in equation (6) may be neglected. Also neglecting terms of order ε, equations (5) and (6) can be substituted into equation (7) to give a single equation for the variation of the temperature, J, along the vertical axis of the cylinder,

$$-\tfrac{4}{3}\cos\theta J' J + \sin^2\theta(J'^2 + \tfrac{2}{3}JJ'') = \tfrac{20}{3}(\tfrac{1}{18})^{1/3} J^{4/3}(\pm J')^{1/3} \quad (8)$$

where the primes indicate differentiation with respect to ξ. In the term on the R.H.S. of equation (8) the positive sign applies for β positive and the negative sign for β negative. The conditions necessary for the existance of a real solution to equation (8), when β is positive, are

$$J = 0; \qquad \xi = 0$$

and

$$JJ'^4 = \tfrac{18}{125}; \qquad \xi = 1.$$

For β negative these conditions are interchanged. With these conditions no additional boundary conditions are required. Equation (8) was solved for the term J on the R.H.S. The resulting equation was then iterated numerically for values of J at twenty points between $0 < \xi < 1$. The resulting function will be discussed in comparison with the experimental results. The value of J on the center line of the cylinder, J_{CL}, which will be required in subsequent calculations was 0·464.

From equations (5) and (6) the parameters F and G are given by,

$$G = 0{\cdot}3816(JJ')^{1/3}$$

and

$$F = -2{\cdot}289\sin\theta(J/J'^2)^{1/3}.$$

Using these expressions and the function J, the boundary-layer thickness and the maximum velocity in the boundary layer at the center line are, respectively,

$$(\delta/D)_{CL} = 3{\cdot}72 Ra^{-1/4}$$

and

$$(vD/a)_{CL} = 0{\cdot}307 Ra^{1/2}.$$

These values will also be compared with experimental observations.

To account for the variation of β with temperature an effective value of β is derived. To do this a polynomial approximation,

$$\frac{\rho_0}{\rho} = 1 + D_1 T + D_2 T^2 + D_3 T^3,$$

to the density of water is used. ρ_0 is the density at 0°C and T is the temperature in degrees Celsius. The fitting parameters obtained in [8] for the temperature range 0 to 20°C are $D_1 = -0{\cdot}6669167 \times 10^{-4}$, $D_2 = 0{\cdot}871689 \times 10^{-5}$, and $D_3 = -0{\cdot}647664 \times 10^{-7}$. The effective β used was arrived at from averaging the buoyancy term in equation (6) over η and ξ. In particular

$$\beta = -\frac{\int_0^1 \int_0^1 \frac{\rho_c - \rho}{\rho} d\eta \, d\xi}{\int_0^1 \int_0^1 \psi \, d\eta \, d\xi}.$$

This form reduces to the normal value of β when density is a linear function of temperature. Carrying out the integral with respect to η using the assumed form of the boundary-layer temperature distribution gives

$$\beta = \beta_1 + \beta_2(T_{CL} - T_w) + \beta_3(T_{CL} - T_w)^2 \quad (9)$$

where

$$\beta_1 = (D_1 + 2D_2 T_w + 3D_3 T_w^2)$$

$$\beta_2 = \tfrac{7}{5}(D_2 + 2D_3 T_w) I_2, \quad \beta_3 = \tfrac{57}{35} D_3 I_3$$

and

$$I_2 = \frac{\int_0^1 \left(\frac{J}{J_{CL}}\right)^2 \mathrm{d}\xi}{\int_0^1 \left(\frac{J}{J_{CL}}\right) \mathrm{d}\xi}, \quad I_3 = \frac{\int_0^1 \left(\frac{J}{J_{CL}}\right)^3 \mathrm{d}\xi}{\int_0^1 \left(\frac{J}{J_{CL}}\right) \mathrm{d}\xi}.$$

In addition if it is assumed that the function J/J_{CL} derived for a constant β is approximately true for a variable β the integrals over ξ can be carried out numerically to give $I_2 = 1{\cdot}215$, and $I_3 = 1{\cdot}627$. The value of the temperature difference, $T_{CL} - T_w$, may then be calculated using β from equation (9) in the Rayleigh number. Thus

$$T_{CL} - T_w = 0{\cdot}464 C_T(|\beta|)^{-1/5} \quad (10)$$

where

$$C_T = \left(\frac{\nu}{a^3 g}\right)^{1/5} DH^{4/5}.$$

The solutions of equation (10) for several values of the parameter C_T are shown in Fig. 2. It will be noted that the solutions form two curves and in the lower temperature range three possible solutions of $T_{CL} - T_w$ exist for a given wall temperature. The dividing line between the two curves is the condition for which β is zero, dashed line in Fig. 2. Thus in the region where three solutions exist the largest corresponds to a small positive β, the middle solution to a small negative β and the smallest solution to a large negative β. These different modes of convection exist because the temperature difference between the core and the wall which drives the convection currents is determined by those same currents.

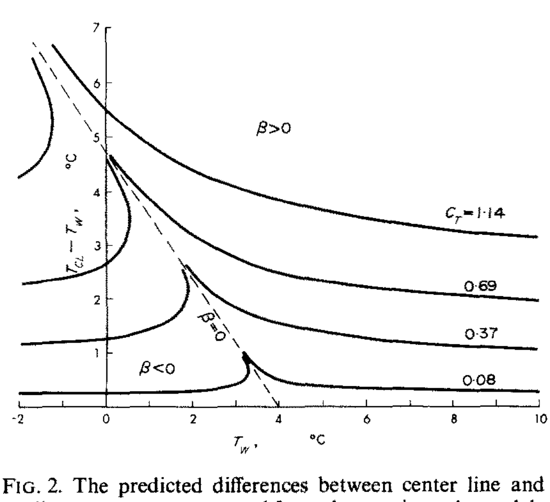


FIG. 2. The predicted differences between center line and wall temperatures computed from the quasi-steady model.

The predictions of this model are however based on a number of approximations which will not be valid close to the line $\beta = 0$. A comparison with experimental measurements will show where the departure from the quasi-steady model occurs.

APPARATUS

Cooling rates were studied in three cylinders of inside diameters 136, 75 and 26 mm. Each was made of copper with approximately 3 mm wall thickness and with a length to diameter ratio of three or greater. A double helix of copper tubing was soldered to the outside of each test cylinder. The cooling fluid, a water–methanol mixture, was circulated in opposing directions through the two helices so as to maintain a uniform temperature over the length of the cylinder. The temperature of the cooling fluid was varied in the required manner using a programmable temperature bath. In these experiments the cylinder was brought to equilibrium at some initial temperature, usually 20°C, and then the temperature was lowered at a constant rate from 0·6 to 54°C/h until ice formed in the cylinder. During the cooling period the cylinder wall temperature could be controlled to within ±0·02°C of the programmed temperature.

The 136-mm dia cylinder was instrumented as shown in Fig. 3 for measurement of temperature distributions, velocity profiles, and flow patterns. Temperature distributions in the water were obtained from copper–constantan thermocouple measurements. Thermocouples were located 2·8 and 23·5 mm from the wall on radii 90° apart, on the center line of the cylinder, and imbedded in the cylinder wall.

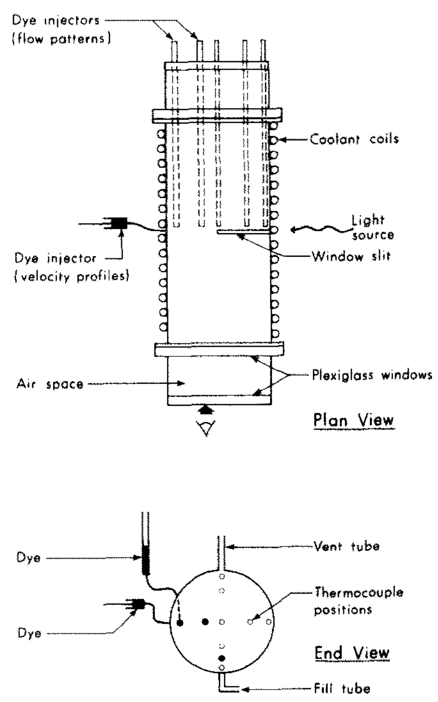


FIG. 3. Cylinder used in experimental determination of temperature distribution, velocity profiles, and flow patterns.

Flow patterns and velocity profiles were observed using a Uranine dye illuminated by a high intensity ultraviolet lamp. This dye has a very intense fluorescence which could be photographed in very low concentrations. The light beam was directed through a 2 mm wide vertical window slit in the side of the tube wall. For flow pattern determination the dye was injected through a number of 1·5 mm dia tubes. To produce the minimum disturbance the dye was injected by gravity flow produced when the reservoir containing the dye was raised by a micrometer screw. The velocity profile across the boundary layer at the horizontal axis of the tube was measured by occasionally injecting a small drop of dye into the cylinder from the side wall. This drop which was injected with a syringe would penetrate the water to about the cylinder centerline leaving a dye trace. Deflection of the dye trace due to convection currents was photographed with time-lapse photography and used to obtain the boundary layer velocity profile. From observations of flow patterns and temperature distributions with and without dye injection it would appear that these injections produce a minimal disturbance to the overall flow.

The 75 and 26 mm dia cylinder were instrumented only for measurement of wall and center line temperature. These small cylinders were used to give an indication of the effect of diameter on the temperature–time history during the inversion process.

RESULTS

A typical time history of the water temperature at the center of the cylinder and the wall temperature is shown in Fig. 4. These results are for the 136 mm cylinder initially at 14·5°C and cooled at a constant rate of 3·8°C/h until ice forms. The heat-transfer behavior may be divided into five regimes. First there is the transient period associated with the commencement of convective flow. Next there is the quasi-steady period in which flow patterns, velocities and the temperature difference between the water and the wall are relatively constant. The third regime begins as the water temperature approaches its maximum density point around 4°C. In this temperature range the convective pattern and the temperature distribution undergoes an inversion. At temperatures below the inversion the convective pattern may again attain a quasi-steady state. The only difference between this and the quasi-steady regime above 4°C is that the entire flow pattern is inverted. The final and fifth regime begins when ice nucleates in the cylinder.

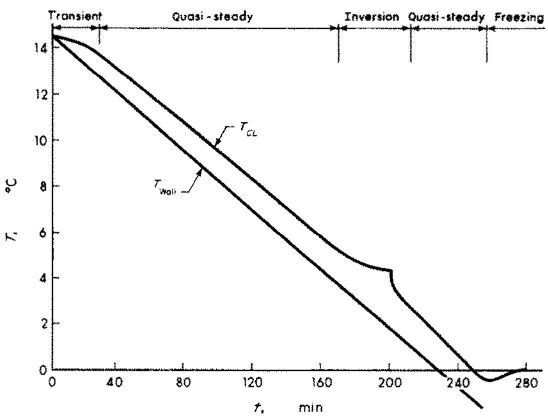


FIG. 4. Typical time history of wall and center time temperatures.

The temperature range over which each of these regimes extend will depend on the rate of cooling, the diameter of the cylinder and the initial temperature of the water.

First the experimental results will be presented for the quasi-steady state above 4°C. The temperatures, velocities and flow patterns existing in this regime will be the starting condition for the inversion that occurs around 4°C. The temperature distributions along the vertical axis are shown in Fig. 5. The temperatures

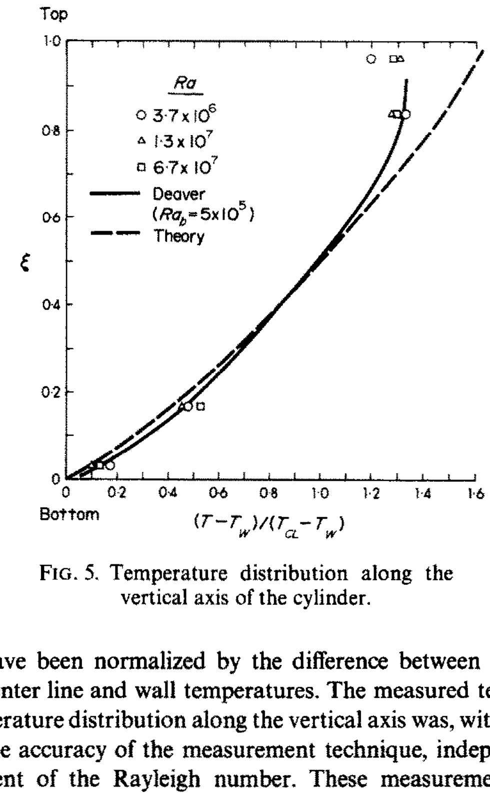


FIG. 5. Temperature distribution along the vertical axis of the cylinder.

have been normalized by the difference between the center line and wall temperatures. The measured temperature distribution along the vertical axis was, within the accuracy of the measurement technique, independent of the Rayleigh number. These measurements agree favourably with the measurements of Deaver [15] even though his results, which were obtained with an interferometer, were limited to $Ra_b < 5 \times 10^5$. Comparing the measured profile with that calculated from the theoretical model it will be observed that there is a significant difference between them near the top of the cylinder. This could relate to the fact that from studies of the flow pattern some unsteady convection was observed in this region.

The temperature profiles indicate the possibility of using a boundary-layer similarity model of the type described in the theory. As a further test of this model the velocity profiles measured across the boundary layer on the horizontal axis of the cylinder were normalized according to the model and plotted in Fig. 6. The velocity profiles in the boundary-layer

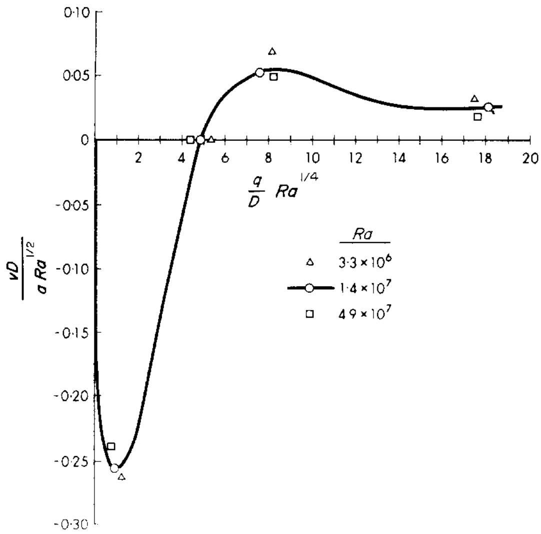


FIG. 6. Normalized velocity profile through the boundary layer on the horizontal axis of the cylinder.

region reduce to one curve independent of Rayleigh number. A region of strong upward flow is observed in the core adjacent to the boundary layer. The analytic model does not account for this region but includes it in the constant velocity core. The predicted boundary-layer thickness nondimensionalized as in Fig. 6 is 3·72. Taking the position of zero velocity as the dividing line between core and boundary layer the measured thickness is approximately 25 per cent greater than calculated. It will be noted that these measurements indicate that the boundary-layer thickness is 10 per cent of the cylinder diameter at a Rayleigh number of 3×10^6. This represents the approximate lower limit of Rayleigh number for which the flow could be considered to be a boundary-layer flow.

The predicted maximum normalized velocity in the boundary layer was 0·307 which is 14 per cent more than observed. However, as is often the case, it will be seen that the integral boundary-layer model will adequately predict overall heat-transfer characteristics in spite of the rather approximate nature of the flow field.

The quasi-steady flow pattern is shown in Fig. 7(a) for a Rayleigh number of 10^7. The flow pattern was observed to be qualitatively the same for all Rayleigh numbers except, as noted in [15], for $Ra > 2 \times 10^7$ an unsteady region of flow exists near the top of the cylinder. In this region the flow consisted of periodic plumes falling counter to the general upward flow in the core.

As the cylinder was cooled through 4°C an inversion of the flow occurred. In Fig. 7 the flow pattern, the temperature profile along the vertical axis, and the velocity profile along the horizontal axis are shown at five different times during the inversion process.

In the quasi-steady regime, Fig. 7(a) the core of the circulation pattern is located above the center line of the cylinder. As the inversion temperature is approached, Fig. 7(b), the circulating flow becomes more symmetrical about the horizontal axis of the cylinder

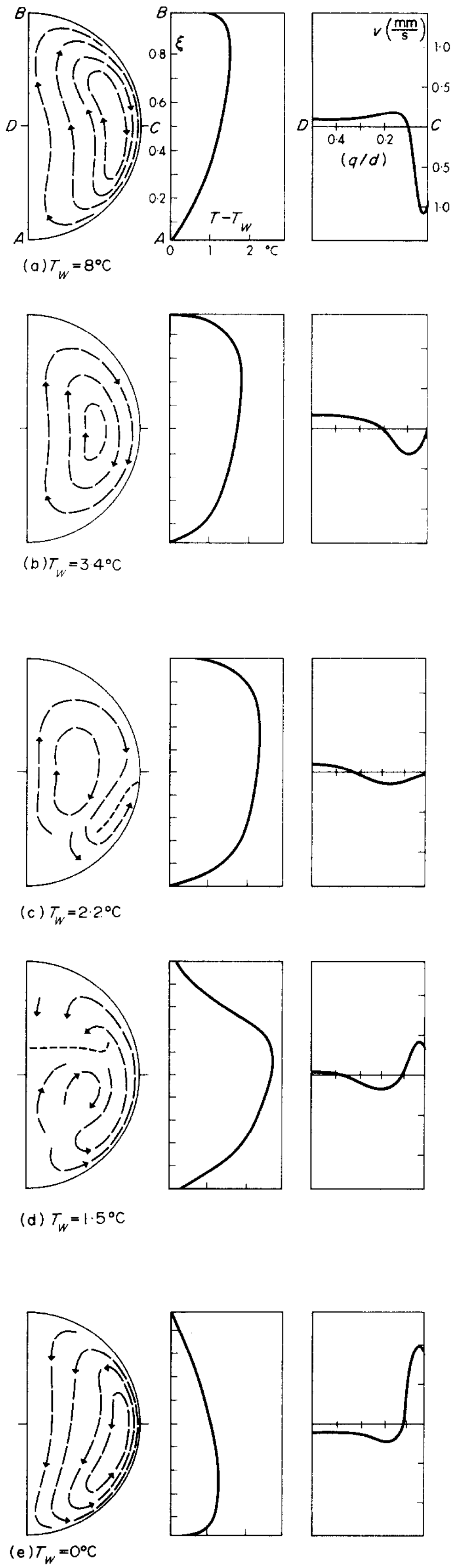


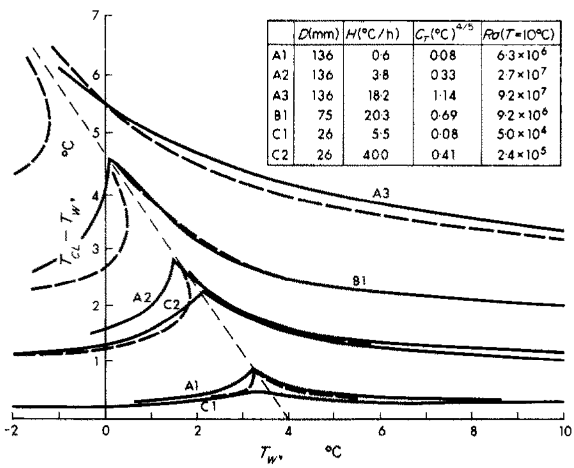
FIG. 7. Flow patterns, temperature distributions, and velocity profiles during the inversion process (136 mm dia cylinder, cooled at 3·8°C/h).

and the velocity boundary-layer becomes thicker. The inversion commences with an upward flow in the boundary layer in the lower third of the cylinder. The dotted lines on the circulation patterns shown in Figs. 7(c) and (d) represents the dividing lines between the reversed and the original flow patterns. In Fig. 7(c) the reverse flow has reached the horizontal axis producing an inflection point in the velocity profile at the cylinder wall. This upward flow continues up the wall forcing the previous circulation to the center of the cylinder. The upward boundary-layer flows from the two halves of the cylinder meet at the top and start the downward flow in the core. The inversion is complete when the downward core flow reaches the bottom of the cylinder. Figures 7(c) and (d) show flow patterns that deviate qualitatively as well as quantitatively from those existing during the quasi-steady period indicating that the quasi-steady model will not apply under these conditions.

During the period when the inversion is taking place velocities are reduced and thus the heat transfer between the water and the cylinder is low. This effect is observed in the fact that the temperatures, $T-T_w$, along the vertical axis increase in Figs. 7(a)–(d). Temperatures start decreasing and the temperature gradient reverses in the core of the cylinder when the flow there reverses, Figs. 7(d) and (e).

The quasi-steady state condition below 4°C is shown in Fig. 7(e). This is similar to the condition in Fig. 7(a) with the top and bottom interchanged.

The qualitative behavior of the flow pattern and temperature and velocity profiles was the same for other cooling rates tried. The temperatures at which the inversion occurred, however, were different. A convenient indicator of inversion is the temperature difference $T_{CL}-T_w$. The measured values of this parameter are shown in Fig. 8 as a function of wall temperature, T_w, for various values of the parameter C_T which correlates different cooling rates and cylinder diameters. Also shown, dashed lines, are the predictions of the quasi-steady model where they depart significantly from the measured values. At $T_w = 10°C$ the values



	D(mm)	H(°C/h)	C_T(°C)$^{4/5}$	Ra(T=10°C)
A1	136	0·6	0·08	$6·3\times10^6$
A2	136	3·8	0·33	$2·7\times10^7$
A3	136	18·2	1·14	$9·2\times10^7$
B1	75	20·3	0·69	$9·2\times10^6$
C1	26	5·5	0·08	$5·0\times10^4$
C2	26	400	0·41	$2·4\times10^5$

FIG. 8. Comparison of the predicted and measured differences $T_{CL}-T_w$ during the inversion process.

of $T_{CL}-T_w$ predicted by quasi-steady model agree within 15 per cent of those measured over the range of diameters and cooling rates used. As T_w is decreased the measured values of $T_{CL}-T_w$ follow the predicted curve for β positive to a point very close to the line (dotted line) for $\beta = 0$. Then the experimental results show a transition occurs to the lower quasi-steady curve for β negative. During the transition the experimental values depart significantly from the quasi-steady results. It is interesting to note however that the deviation from the prediction of the quasi-steady model occurs only for a relatively small range to temperature. Also it will be noted from comparing the experimental curves labelled A and C that the parameter C_T which correlates results in the quasi-steady regime is not the only parameter of significance during the transition period. In fact the curve $C1$ shows only a very small increase in the temperature difference, $T_{CL}-T_w$, during the transitions. This difference between the behavior of the large and the small cylinders may be attributable to the fact that at a given temperature difference the Rayleigh number of the small cylinder is much smaller than that for the large cylinder. At the smaller Rayleigh numbers that exist for the small cylinders conduction heat transfer which is not effected by the density inversion, becomes an important contributor to the overall heat transfer mechanism.

It will be noted in Fig. 8 that experimental results of water temperatures exist at temperatures below 0°C. In the experiments supercooling of the water occurred to temperatures as low as −7°C before ice nucleation occurs. The nucleation and growth of ice in a horizontal cylinder is the subject of continuing experimental investigations.

CONCLUSIONS

When the wall of a horizontal cylinder of fluid is cooled at a constant rate the free convection pattern in the cylinder will attain a quasi-steady state after some initial transient period. In the quasi-steady state velocities, flow patterns, and temperature differences between the fluid and wall will be constant in time. For Rayleigh numbers greater than about 10^6 the flow in the cylinder can be approximated by distinct core and boundary-layer flows. Under these conditions an analytic model using an integral treatment of the boundary layer adequately predicts core temperatures and, to a lesser degree of accuracy, boundary-layer thickness and velocity.

In the region of the maximum density of water the thermal coefficient of expansion approaches zero and the Rayleigh number as normally written becomes inadequate to define the problem. In this region an averaged buoyance force in the cylinder was used to define an effective coefficient of expansion. The quasi-steady model using this value then predicts that three possible values of the core temperature may exist at some values of wall temperature below 4°C. These different values are associated with different modes of free convection in the cylinder. Physically the different modes arise because the difference between the core and the wall temperature which drives the convection currents is also determined by them.

Experimental measurements of the core temperature as the cylinder is cooled through the maximum density point suggest that the inversion of the flow pattern occurs by a transition from one quasi-steady mode of convection to another. Detailed observations of the temperature distribution, velocity profiles, and flow patterns during the transition show that the inversion begins with a reversal of the flow direction in the boundary layer near the bottom of the cylinder. This reversal spreads to the top of the cylinder and then begins the reverse flow in the core. The period during which this inversion occurs is a complex transient phenomenon which cannot be treated by the quasi-steady model. Also it was noted that because of the decrease in Rayleigh number that occurs near 4°C, heat conduction in the core flow can have a significant effect on the core temperature during the inversion.

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REFROIDISSEMENT D'UN CYLINDRE HORIZONTAL D'EAU AUTOUR DE SON POINT DE DENSITE MAXIMALE A 4°C

Résumé—On étudie la convection naturelle dans un cylindre horizontal refroidi de façon constante dans le temps. Les distributions de température, profils de vitesse dans la couche limite et structure d'écoulement ont été mesurés et comparés aux prévisions d'un modèle qui a également été développé pour la couche limite quasi-stationnaire. Le comportement de la convection libre dans le cas où le cylindre est refroidi jusqu'au point de densité maximale de l'eau est d'un intérêt tout particulier pour cette étude. Le modèle théorique prévoit l'existence de trois modes de convection quasi-stationnaires à des températures inférieures à 4°C. L'inversion de l'écoulement observée expérimentalement fait intervenir une transition d'un mode quasi-stationnaire à un autre.

ABKÜKLUNG EINES WAAGERECHTEN ZYLINDERS MIT WASSER VON MAXIMALER DICHTE BEI 4°C

Zusammenfassung—Es wurde die freie Konvektion auf der Innenseite eines waagerechten Rohres bei zeitlich konstanter Kühlung untersucht. Temperaturverteilung, Grenzschichtgeschwindigkeitsprofile und Stromlinienbilder wurden gemessen und mit den theoretischen Werten eines ebenfalls entwickelten Modells verglichen. Von besonderem Interesse bei diesen Untersuchungen war das Verhalten der freien Konvektion, wenn der Zylinder durch Wasser maximaler Dichte abgekühlt wurde. Aus dem theoretischen Modell läßt sich das Vorhandensein von drei quasistationären Arten der Konvektion bei einigen Temperaturen unter 4°C ableiten. Die Umkehrung der experimentell ermittelten Stromlinienbilder macht die Überführung von einem quasistationären Zustand zum anderen kompliziert.

ОХЛАЖДЕНИЕ ГОРИЗОНТАЛЬНОГО ЦИЛИНДРА ВОДОЙ В ТОЧКЕ МАКСИМАЛЬНОЙ ПЛОТНОСТИ ПРИ 4°C

Аннотация — Исследовалась свободная конвекция в горизонтальном цилиндре, охлаждаемом водой с постоянной во времени скоростью. Измерялись распределения температур, профили скорости в пограничном слое и картины течения. Полученные данные сравнивались с расчетами для модели развивающегося квазистационарного пограничного слоя. Особый интерес для этого исследования представлял процесс свободной конвекции, так как охлаждение цилиндра производилось при максимальной плотности воды. Теоретическая модель предсказывает существование трех квазистационарных режимов конвекции для температур ниже 4°C. При обратной картине течения, наблюдаемой экспериментально, происходит смена квазистационарного режима.